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Algebraic Nature of Fuzzy Subgroups Under Homomorphism

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ABSTRACT
The purpose of this paper is to discuss the nature of fuzzy level subgroups under homomorphism and anti –homomorphism. It is shown that homomorphic image (pre-image)/anti-homomorphic image (pre-image) of level subgroups is also a level subgroup.

Keywords
Fuzzy homomorphism, fuzzy anti-homomorphism, fuzzy level subgroups, homomorphic image, anti homomorphic image

1. INTRODUCTION
Fuzzy set was first introduced by Zadeh. Rosenfield introduced the notion of fuzzy subgroups.

The effect of group homomorphism on fuzzy groups was studied by Rosenfeld, Anthony and Sherwood, Sidky and Mishref and Akgul.Rosenfeld proved that if f is a group homomorphism on G, then

S (G) denotes the set of all fuzzy sets defined on G

S (f (G)) denotes the set of all fuzzy sets defined on f (G)

Provided F has $\vee$ -property and

Later Anthony and Sherwood observe that in the first case the restriction "F has $\vee$-property" is redundant. Sidky and Mishref proved that if

$f : G \rightarrow G^1$

f *is* a group homomorphism and *F* is a fuzzy subgroup of G "with respect to a continuous t-norm T" then f (F) is a fuzzy subgroup of G^1 with respect to T.

Since, $\wedge$ is a continuous t-norm it follows that,

f (F) belongs to S(G^1) whenever F belongs to S(G)

It was proved by Agkul that

F^1 belongs to S (G^1) implies that f^{-1} belongs to S(G)

In this chapter we study the effect of group homomorphism on the level subgroups of fuzzy groups.

2. PRELIMINARIES

2.1 Group Homomorphism
If (G, .) and (G^1, .) any two groups, then the function f is called a group homomorphism if.

$f(xy) = f(x) f(y), \forall x, y \in G$

2.2 Group Anti-Homomorphism
If (G , .) and (G^1, .) any two groups, then the function f is called a group anti-homomorphism if .

$f(xy) = f(y) f(x), \forall x, y \in G$

2.3 Image of a fuzzy set and pre-image of a fuzzy set
Suppose S is a groupoid and $f : S \rightarrow I$ is a fuzzy set and $\phi : S \rightarrow S$ is a mapping and $g : \phi(S) \rightarrow I$ is a fuzzy set defined by

$$g(y) = \max_{x \in \phi^{-1}(y)} \{ f(x) \}$$

Then g is called image of f under ϕ.

Conversely, f is called pre-image of g under ϕ .

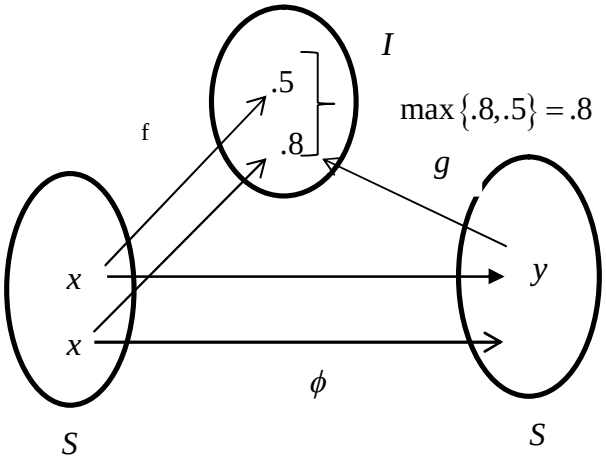

Figure 1: Image and Pre-Image of fuzzy set

3. SOME PROPOSITIONS

3.1 Proposition 1

The homomorphic image of a level subgroup of a fuzzy subgroup of a group G is a level subgroup of a fuzzy subgroup of a group G^1.

Proof:

Let G and G^1 be any two groups.

Let $f : G \to G^1$ be a homomorphism.

That is

$$f(xy) = f(x) f(y), \forall x, y \in G$$

Let, $V = f(A)$ where A is a fuzzy subgroup of a group G.

Clearly V is a fuzzy subgroup of a group G^1.

Let

$x, y \in G$

Implies $f(x)$ and $f(y)$ in G^1.

Clearly A_t is a level subgroup of A.

That is $A(x) \geq t$ and $A(y) \geq t$;

$A(xy^{-1}) \geq t$.

We have to prove that $f(A_t)$ is a level subgroup of V.

Now

$V(f(x)) \geq A(x) \geq t \Rightarrow V(f(x)) \geq t$;

$V(f(y)) \geq A(y) \geq t \Rightarrow V(f(y)) \geq t$

And $V\left(f(x)(f(y))^{-1}\right) = V\left(f(x) f(y^{-1})\right)$, as f is a homomorphism

$= V\left(f(xy^{-1})\right)$, as f is a homomorphism.

$\geq A(xy^{-1}) \geq t$

Which implies that $V\left(f(x)(f(y))^{-1}\right) \geq t$

Hence $f(A_t)$ is a level subgroup of a fuzzy subgroup V of a group G^1.

3.2 Proposition 2

The homomorphic pre-image of a level subgroup of a fuzzy subgroup of a group G^1 is a level subgroup of a fuzzy subgroup of a group G.

Proof:

Let G and G^1 be any two groups.

Let $f : G \to G^1$ be a homomorphism.

That is $f(xy) = f(x) f(y), \forall x, y \in G$

Let, $V = f(A)$ where V is a fuzzy subgroup of a group G^1.

Clearly A is a fuzzy subgroup of group G.

Let $f(x), f(y) \in G^1$, implies x and y in G.

Clearly $f(A_t)$ is a level subgroup of V.

That is $V(f(x)) \geq t$ and $V(f(y)) \geq t$;

$V\left(f(x)(f(y))^{-1}\right) \geq t$.

We have to prove that A_t is a level subgroup of A.

Now, $A(x) = V(f(x)) \geq t \Rightarrow A(x) \geq t$

$A(y) = V(f(y)) \geq t \Rightarrow A(y) \geq t$; And

$A(xy^{-1}) = V\left(f(xy^{-1})\right)$

$= V\left(f(x) f(y^{-1})\right)$, as f is a homomorphism

$= V\left(f(x)(f(y))^{-1}\right)$, as f is a homomorphism

$\geq t$,

Which implies that $A(xy^{-1}) \geq t$.

Hence A_t is a level subgroup of a fuzzy

3.3 Proposition 3

The anti-homomorphic image of a level subgroup of a fuzzy subgroup of a group G is a level subgroup of a fuzzy subgroup of a group G^1.

Proof:

Let G and G^1 be any two groups.

Let $f : G \to G^1$ be a anti-homomorphism.

That is $f(xy) = f(y) f(x), \forall x, y \in G$

Let, $V = f(A)$ where A is a fuzzy subgroup of a group G.

Clearly V is a fuzzy subgroup of a group G^1.

Let $x, y \in G$, implies $f(x)$ and $f(y)$ in G^1.

Clearly A_t is a level subgroup of A.

That is $A(x) \geq t$ and $A(y) \geq t$;

$A(y^{-1}x) \geq t$

We have to prove that $f(A_t)$ is a level subgroup of V.

Now $V(f(x)) \geq A(x) \geq t \Rightarrow V(f(x)) \geq t$;

$V\left(f\left(y\right)\right) \geq A\left(y\right) \geq t \Rightarrow V\left(f\left(y\right)\right) \geq t$

And $V\left(f\left(x\right)\left(f\left(y\right)\right)^{-1}\right) = V\left(f\left(x\right) f\left(y^{-1}\right)\right)$,as f is an anti-homomorphism

$= V\left(f\left(y^{-1}x\right)\right)$ as f is an anti-homomorphism

$\geq A\left(y^{-1}x\right) \geq t$

Which implies that

$V\left(f\left(x\right)\left(f\left(y\right)\right)^{-1}\right) \geq t$

Hence $f(A_t)$ is a level subgroup of a fuzzy subgroup V of a group G^1.

3.4 Proposition 4

The anti-homomorphism pre-image of a level subgroup of a fuzzy subgroup of a group G^1 is a level subgroup of a fuzzy subgroup of a group G.

Proof:

Let G and G^1 be any two groups.

Let $f : G \rightarrow G^1$ be a anti-homomorphism.

That is

$f\left(xy\right) = f\left(y\right) f\left(x\right), \forall x, y \in G$

Let,

$V = f\left(A\right)$ Where V is a fuzzy subgroup of a group G^1

Clearly A is a fuzzy subgroup of group G

Let $f\left(x\right), f\left(y\right) \in G^1$, implies x and y in G.

Clearly $f\left(A_t\right)$ is a level subgroup of V.

That is $V\left(f\left(x\right)\right) \geq t$ and $V\left(f(y)\right) \geq t$

$V\left(\left(f\left(y\right)\right)^{-1} f\left(x\right)\right) \geq t$.

We have to prove that A_t is a level subgroup of A.

Now, $A\left(x\right) = V\left(f\left(x\right)\right) \geq t \Rightarrow A\left(x\right) \geq t$

$A\left(y\right) = V\left(f\left(y\right)\right) \geq t \Rightarrow A\left(y\right) \geq t$; And

$A\left(xy^{-1}\right) = V\left(f\left(xy^{-1}\right)\right)$

$= V\left(f\left(y^{-1}\right) f\left(x\right)\right)$, as f is a anti-homomorphism

$= V\left(\left(f\left(y\right)\right)^{-1} f\left(x\right)\right) \geq t$, as f is a anti-homomorphism

Which implies that $A\left(xy^{-1}\right) \geq t$.

Hence A_t is a level subgroup of a fuzzy subgroup A of a group G.

4. CONCLUSION

This study helps us to find out the nature of group homomorphism on the chains of level subgroups of fuzzy subgroups, on the T-Fuzzy subgroups, TL-Fuzzy subgroups etc.

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